Wax

22042

ARPA Network Information Center
Stanford Research Institute
Menlo Park, California 94025

A. K. Jain (Collins)

22 February 1974

Packet Radio Temporary Note # 93

NIC # 22042

CRITERIA OF CODE SELECTION IN MSK SENSE

1.0 GENERAL.

The MSK (Minimum Shift Keying) type of modulation utilizes a single physical channel consisting of a pair of equivalent quadrature channels. The practical implementation of MSK essentially involves transmitting two separate signals one chip (T_c) apart (assuming code spread) on the individual quadrature subchannel. Each signal is cosine weighted* over every two-chip interval on each subchannel. The resultant signal is phase continuous at each transition.

The MSK signal can either utilize a single code for each bit with alternating chips transmitted on the two orthogonal subchannels, or it can utilize two independent 1 bit length codes staggered by one chip interval on individual channels. In order that the matched filters (SAWD's) may be implemented for detection of signal corresponding to each subchannel, it is recommended that two codes of the same general class of family (e.g., maximal length, Golay's complementary, etc.) be used for the two subchannels. This ensures that the inherent autocorrelation properties of each code class will be undisturbed.

---

*Several different weighting functions can be used depending on the desired signal structure.

Received at NIC 1-FEB-74

For example, a maximal length code results into -1 side lobes when sequentially repeated. This would not occur with one maximal length code with its alternate chips transmitted on two subchannels and detected through independent matched filters on each subchannel. Similarly, in order to retain the property of Golay's complementary sequences, it is desirable that a complementary pair of codes be transmitted on the two subchannels, so that the sum of the autocorrelation side lobes is zero for all non-zero shifts.

Unless an additional constraint can be implemented upon the signal design, there is a cross-coupling between the two subchannels. One purpose of this paper is to identify mathematically the time and phase relationship between the various autocorrelating and cross-correlating terms in the MSK application, so they can be combined properly to indicate the output of the detector at each chip interval. This relationship is derived in Section 2.0.

Based on the time relationship between the autocorrelation and cross-correlation terms at any chip instant, the criteria for selecting the codes are outlined. The impact of dual data rate on the code selection is also considered.

2.0 A MATHEMATICAL MODEL OF MSK MODEM

Figure 1 depicts a model of MSK modem that has been used to establish time and phase relationship between the autocorrelating and cross-correlating terms. The effect of the cosine weighting function is disregarded in this model because the results of the model are to be used only to develop a time correlating technique on a chip-by-chip basis. However, since the auto-correlation or cross-correlation between individual subchannels

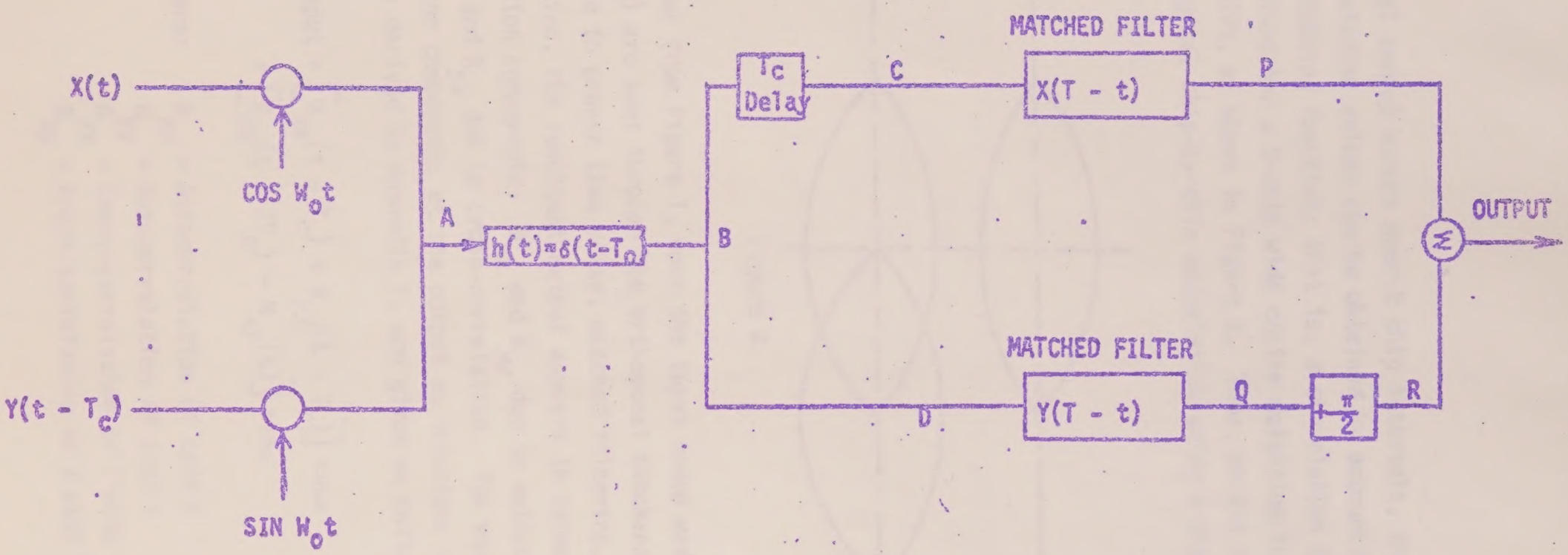
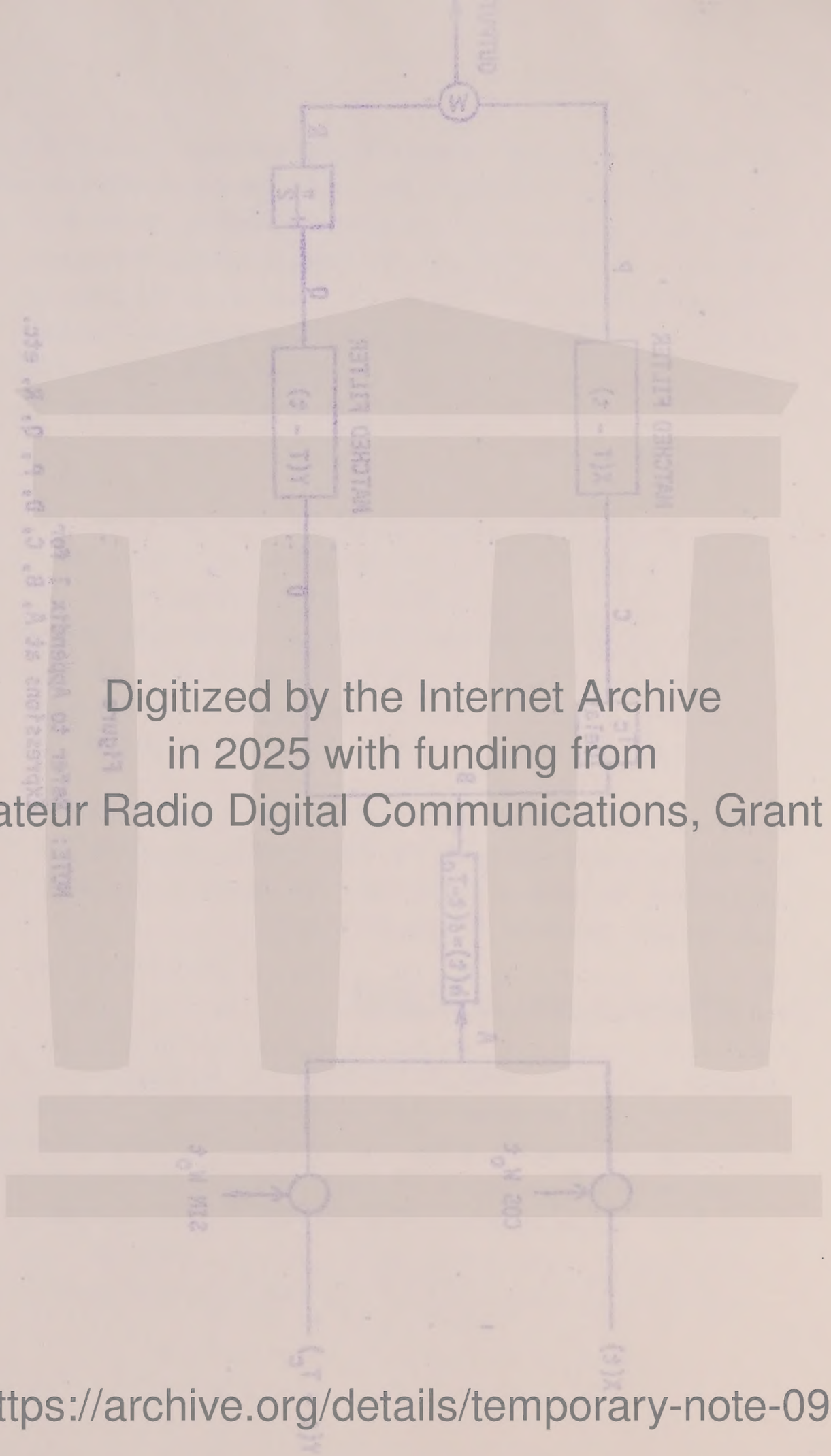


Figure 1

NOTE: Refer to Appendix 1 for Expressions at A, B, C, D, P, Q, R, etc.



Digitized by the Internet Archive
in 2025 with funding from
Amateur Radio Digital Communications, Grant 151

https://archive.org/details/temporary-note-095

(in digital sense) occurs over 2 chip intervals, the in-between chip correlation values can be obtained by accounting for the cosine weighting function; that is, a correlation factor can be determined for a 2-chip wide cosine weighting function for 1 chip shift, as shown in Figure 2. Thus, we are able to perform a digital chip-by-chip correlation using a digital computer program.

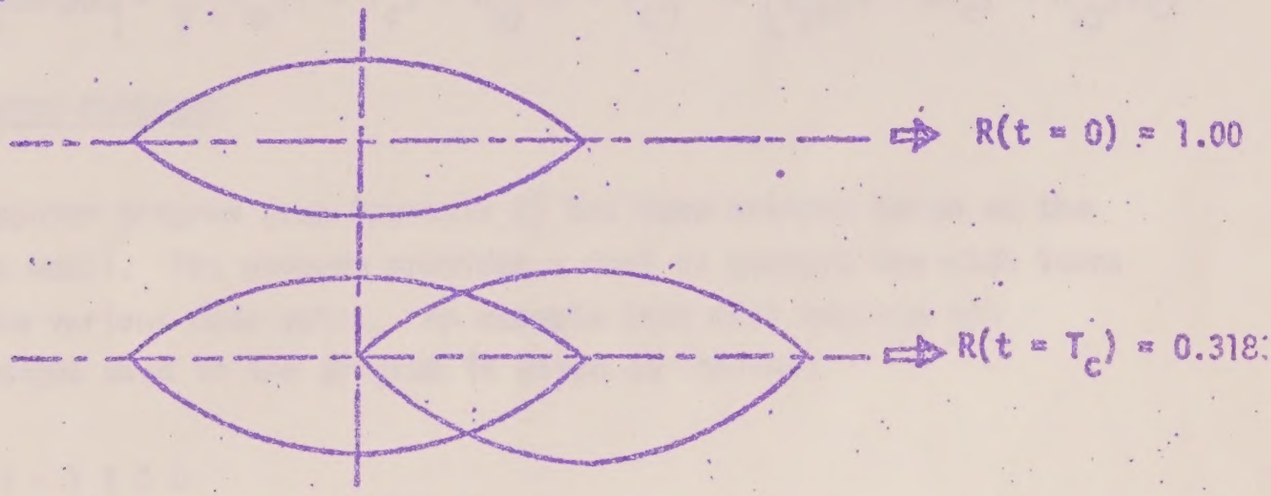


Figure 2

As obvious from Figure 1, once the input coded waveforms X(t), and $Y(t - T_c)$ are sent through the orthogonal subchannels, and subjected to proper time delay, matched filtering, and phase translation, the resultant output appears in terms of four correlation components: R_{xx} and R_{yy} due to autocorrelations, and R_{xy} and R_{yx} due to cross-correlation. The equation relating these four components at the output as functions of time and phase is derived in Appendix 1, and given as follows:

$$\text{Output} = \left[R_{xx}(t - T_c) + R_{yy}(t - T_c)\right] \cos\alpha + \left[R_{yx}(t - 2T_c) - R_{xy}(t)\right] \sin\alpha$$

Where:
R_{xx} = Autocorrelation of code X
R_{yy} = Autocorrelation of code Y
R_{yx} = Cross-correlation of Y with X
R_{xy} = Cross-correlation of X with Y

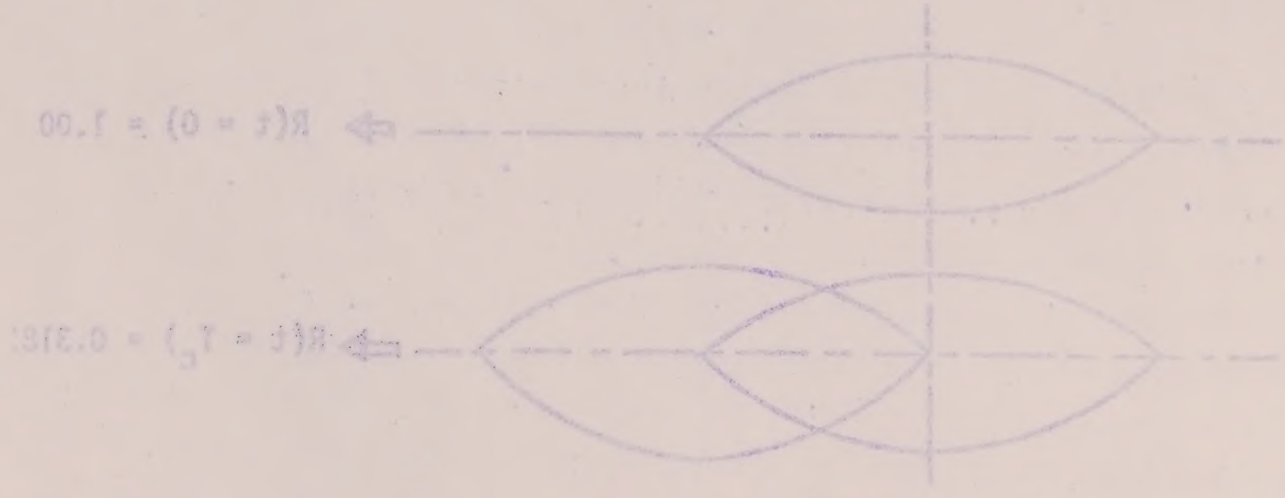


Figure 2

T_c = One chip interval

$\alpha = W_o(t - T_o)$

$W_o = 2\pi f_o$

T_o = Delay in the channel

f_o = Integer multiple of chip frequency

$$|\text{Output}| = \sqrt{\left[R_{xx}(t - T_c) + R_{yy}(t - T_c)\right]^2 + \left[R_{yx}(t - 2T_c) - R_{yy}(t)\right]^2}$$

2.1 Computer Program:

A computer program (see Appendix 2) has been written based on the above model. The program provides a tool to compare the side lobes due to various code pairs. An example that will explain the technique used in the program is given as follows:

Let X = 1 1 1 0

Y = 1 1 0 1

Then, the correlation terms every two chips are:

R_{xx} = -1 0 1 4 1 0 -1

R_{xy} = -1 0 3 0 1 0 1

R_{yy} = 1 1 -1 4 -1 0 1

R_{yx} = 1 0 1 0 3 0 -1

Using the correlation factor of 0.3183 discussed in Section 2, the in-between chip correlation terms can be filled in to give the chip-by-chip correlation given as follows:

T_c = One chip interval

$\epsilon = W_o(t - T_o)$

$W_o = 2\pi f_o$

T_o = Delay in the channel

f_o = Integer multiple of chip frequency

$$\text{output} = \sqrt{\left[R_{xx}(t - T_c) + R_{yy}(t - T_c)\right]^2 + \left[R_{yx}(t - 2T_c) - R_{xy}(t)\right]^2}$$

2.1 Computer Program:

A computer program (see Appendix 2) has been written based on the above model. The program provides a tool to compare the side lobes due to various code pairs. An example that will explain the technique used in the program is given as follows:

Let X = 1 1 1 0

Y = 1 1 0 1

Then, the correlation terms every two chips are:

R_{xx} = -1 0 1 4 1 0 -1

R_{xy} = -1 0 3 0 1 0 1

R_{yy} = 1 1 -1 4 -1 0 1

R_{yx} = 1 0 1 0 3 0 -1

Using the correlation factor of 0.3183 discussed in Section 2, the in-between chip correlation terms can be filled in to give the chip-by-chip correlation given as follows:

R_{xx} = -.3 -1 -.3 0 .3 1 1.5 4 1.5 1 .3 0 -.3 -1

R_{xy} = -.3 -1 -.3 0 .9 3 .9 0 .3 1 .3 0 .3 1

R_{yy} = .3 1 .3 0 -.3 -1 .9 4 .9 -1 -.3 0 .3 1

R_{yx} = .3 1 .3 0 .3 1 .3 0 .9 3 .9 0 -.3 -1

Now, according to the output equation of Section 2, R_{xx} and R_{yy} terms are slided to the right by 1 chip, and R_{yx} terms are slided by two chip, shown as follows:

R_{xx} = X -.3 -1 -.3 0 .3 1 1.5 4 1.5 1 .3 0 -.3 -1 -.3 0

R_{xy} = -.3 -1 -.3 0 .9 3 .9 0 .3 1 .3 0 .3 1 .3 0 0

R_{yy} = X .3 1 .3 0 -.3 -1 .9 4 .9 -1 -.3 0 .3 1 .3 0

R_{yx} = X X .3 1 .3 0 .3 1 .3 0 .9 3 .9 0 -.3 -1 -.

Then,

$(R_{xx} + R_{yy})$ = X 0 0 0 0 0 0 2.4 8 2.4 0 0 0 0 0 0 0

$(R_{yx} - R_{xy})$ = .3 1 .6 1 -.6 -3 -.6 1 0 -1 .6 3 .6 -1 -.6 -1 -.3

Output = .3 1 .6 1 -.6 -3 -.6 2.8 8 2.8 .6 3 .6 -1 -.6 -1 -.3

Notice that the autocorrelation in MSK is four chip wide.

Two more test examples are listed (as the program outputs) in Appendix 2. Based on this technique several code pairs consisting of maximal length class, or Golay's sequences will be studied to select the desirable code pairs. The criteria of their selection are outlined in the following section.

3.0 CRITERIA OF CODE SELECTION

Based on the result of Section 2.0, the following ground rules are applicable for selecting code pairs for M.S.K. applications:

A. Ideally, the absolute value of side lobes as a result of auto-correlation and cross-correlation of two codes should go to zero taking into account the proper time shifts described in the above section. Practically, however, they should be minimized.

B. For Golay's pairs, specifically, the only important criterion is that the difference of the cross-correlation terms staggered by 2 chips should result in minimum side lobes. This is because the side lobes due to autocorrelation are always zero.

C. With maximal length code pairs, the property that the auto-correlation side lobes are always -1 after the code is sequentially repeated is ruined due to the cross-correlation side lobe. However, this should not have a significant impact in the non-overlapping packet environment once we could attain synch to a starting pulse. Therefore, the main consideration with maximal length code pairs for non-overlapping packets is to minimize the side lobes during the time the first bit is propagated through the matched filters. During this interval, it is the overall impact of the output equation of Section 2.0 that has to be accounted for in minimizing the side lobes.

D. With maximal length code pairs in the overlapping packet environment (when multiple detectors are used) the side lobes have to be minimized not only during the first bit interval but during all bit intervals so that the sum of the correlations due to the interfering packets is minimum.

E. Since dual data rate is to be used in the ARPA system, it is important to take into account the cross-correlation between Hi-Lo rate codes. For example, if 128 chips/bit complementary sequence is used for the low rate, and 32 chips/bit complementary is used for the high rate, then it is important to determine in advance that the 32 chip SAWD will not result in high cross-correlations when 128 chip code is propagated through, and vice versa.

APPENDIX 1

Refer to Figure 1 in Section 2.0 of this paper.

$$A = X(t)\cos W_o t + Y(t - T_c)\sin W_o t$$

$$B = X(t - T_o)\cos W_o(t - T_o) + Y(t - T_c - T_o)\sin W_o(t - T_o)$$

$$D = B$$

$$C = X(t - T_o - T_c)\cos W_o(t - T_o - T_c) + Y(t - 2T_c - T_o)\sin W_o(t - T_o - T_c)$$

Now let $W_o = nW_c$ where W_c is the chip frequency.

$$= \frac{2\pi n}{T_c}$$

$$\therefore \quad \cos W_o(t - T_o - T_c) = \cos \frac{2\pi n}{T_c}(t - T_o - T_c)$$

$$= \left\{\cos \frac{2\pi n(t - T_o)}{T_c} - 2\pi n\right\}$$

$$= \left\{\cos 2\pi n - \frac{2\pi n(t - T_o)}{T_c}\right\}$$

$$= \cos\left\{\frac{2\pi n(t - T_o)}{T_c}\right\} \quad \text{for all integer values of n}$$

$$= \cos W_o(t - T_o)$$

$$= \cos\alpha$$

Similarly, $\sin W_o(t - T_o - T_c) = \sin W_o(t - T_o)$ for all n.

$$= \sin\alpha$$

Hence, $C = X(t - T_o - T_c)\cos\alpha + Y(t - 2T_c - T_o)\sin\alpha$

$D = X(t - T_o)\cos\alpha + Y(t - T_c - T_o)\sin\alpha$

Then, through matched filters, we get

$$P = R_{xx}(t - T_o - T_c - T + t)\cos\alpha + R_{yx}(t - 2T_c - T_o - T + t)\sin\alpha$$

$$Q = R_{xy}(t - T_o - T + t)\cos\alpha + R_{yy}(t - T_c - T_o - T + t)\sin\alpha$$

$$R = R_{xy}(t - T_o - T + t)\cos(\alpha + \pi/2) + R_{yy}(t - T_c - T_o - T + t)\sin(\alpha + \pi/2)$$

$$= R_{yy}(t - T_c - T_o - T + t)\cos\alpha - R_{xy}(t - T_o - T + t)\sin\alpha$$

$$\text{Output} = P + R$$

$$= \left\{R_{xx}(t - T_o - T_c - T + t) + R_{yy}(t - T_c - T_o - T + t)\right\}\cos\alpha$$

$$+ \left\{R_{yx}(t - 2T_c - T_o - T + t) - R_{xy}(t - T_o - T + t)\right\}\sin\alpha$$

$$= \left\{R_{xx}(t - T_c) + R_{yy}(t - T_c)\right\}\cos\alpha + \left\{R_{yx}(t - 2T_c) - R_{xy}(t)\right\}\sin\alpha$$

APPENDIX 2

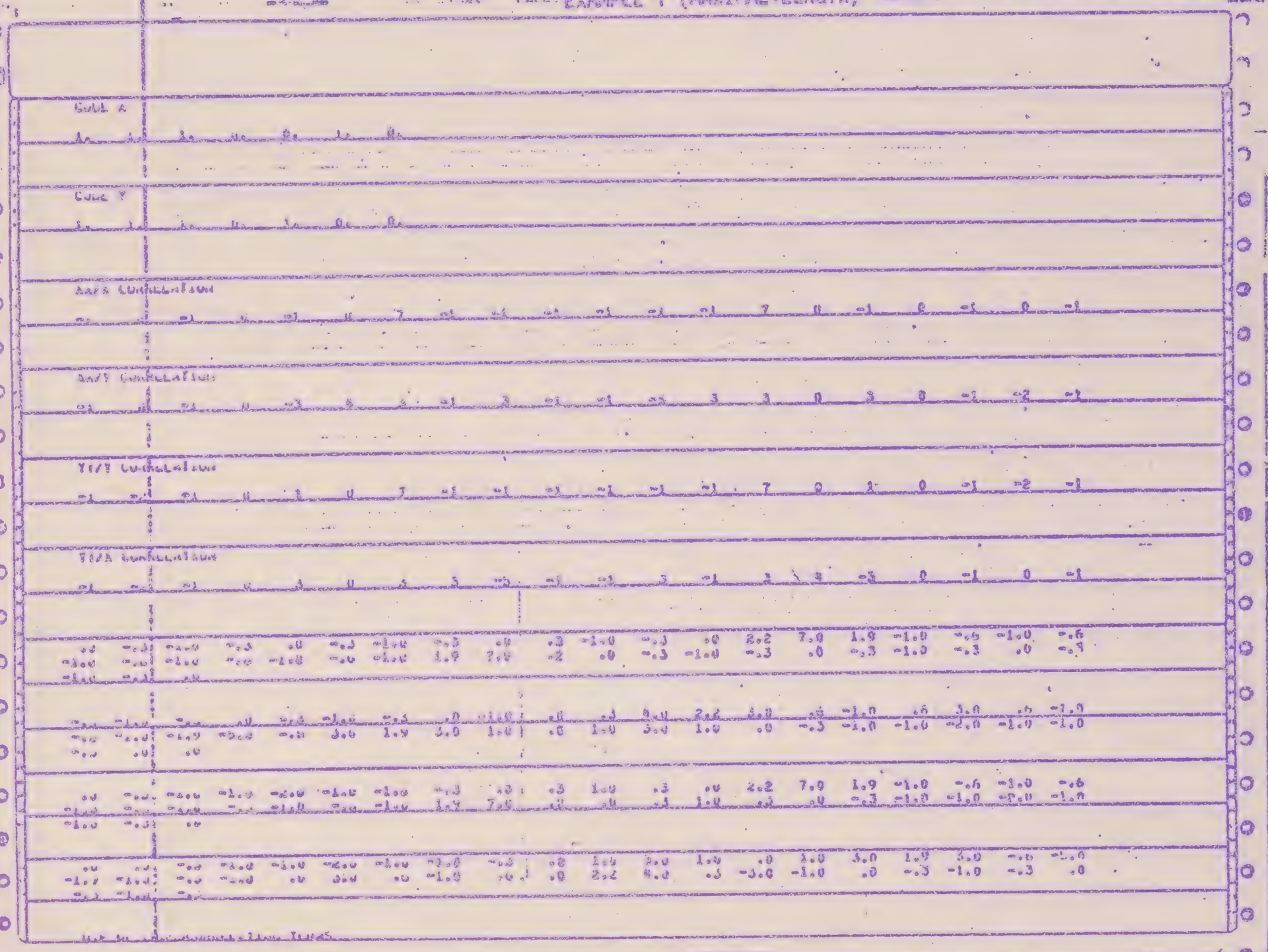
This appendix lists a program that calls for a subroutine "COREL" listed in Packet Radio Note #73. Two examples are presented at the end. First example is for a 7-length maximal length code pair, and the second example is for an 8-length Golay's complementary pair.

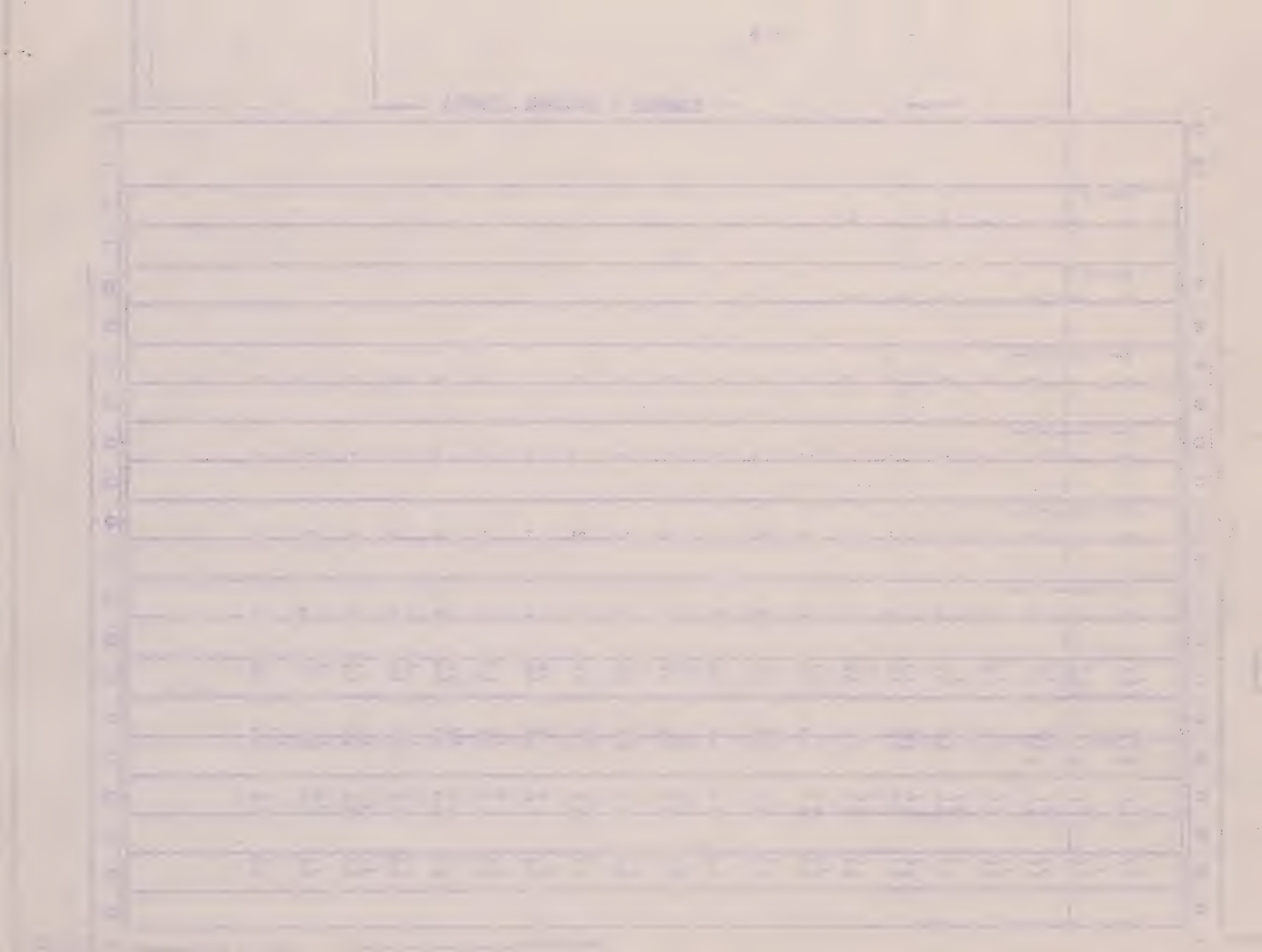
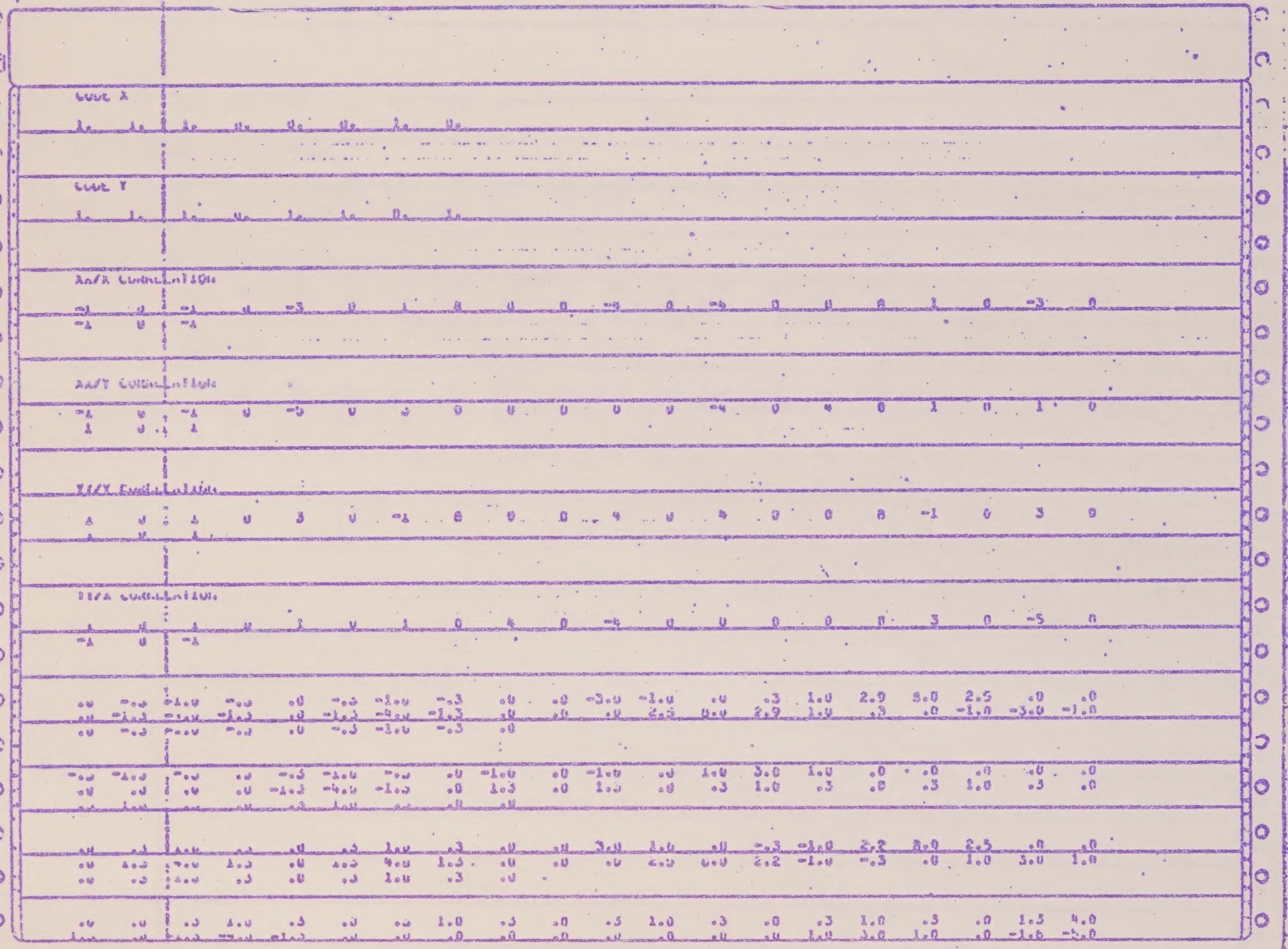
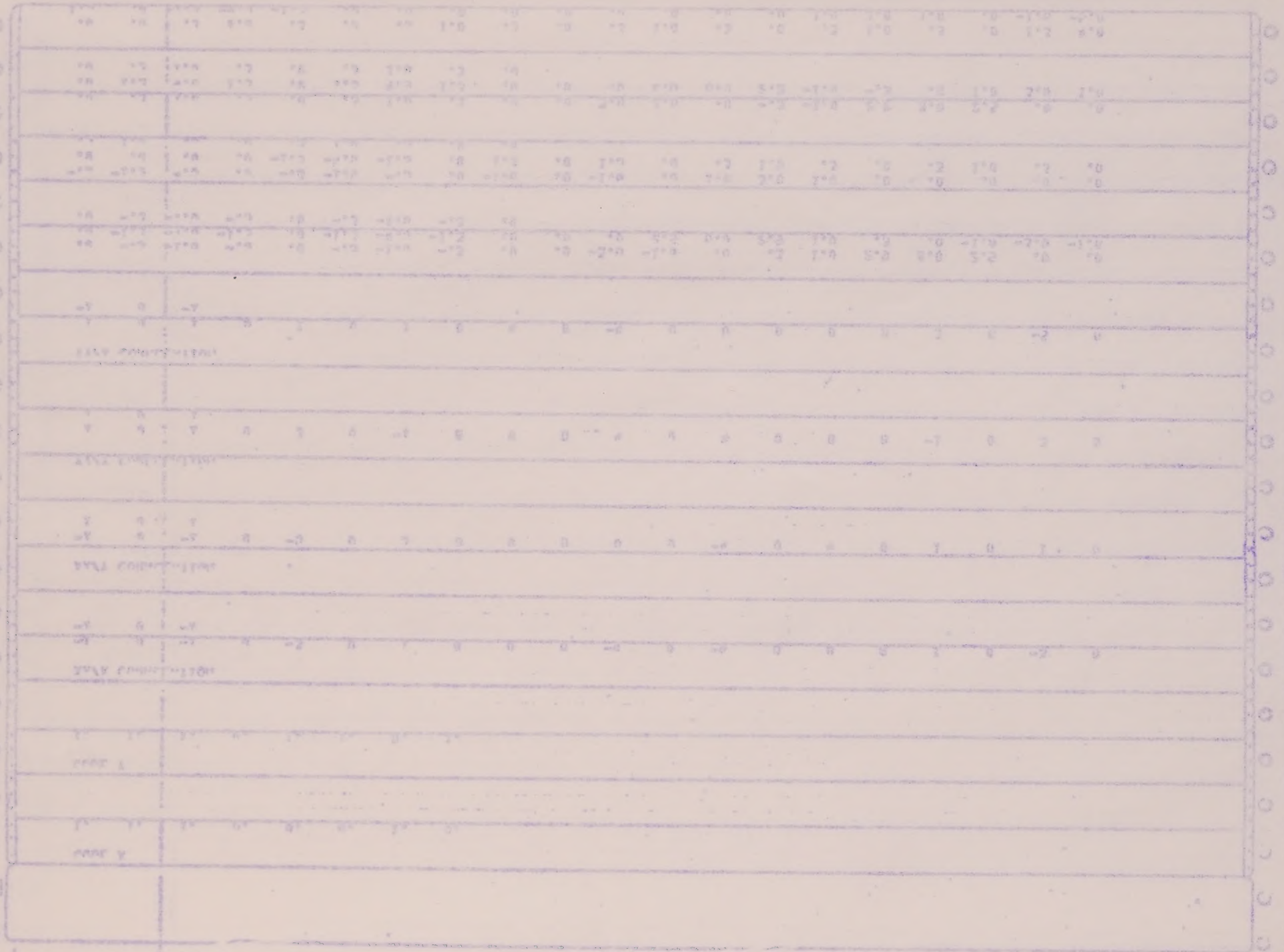
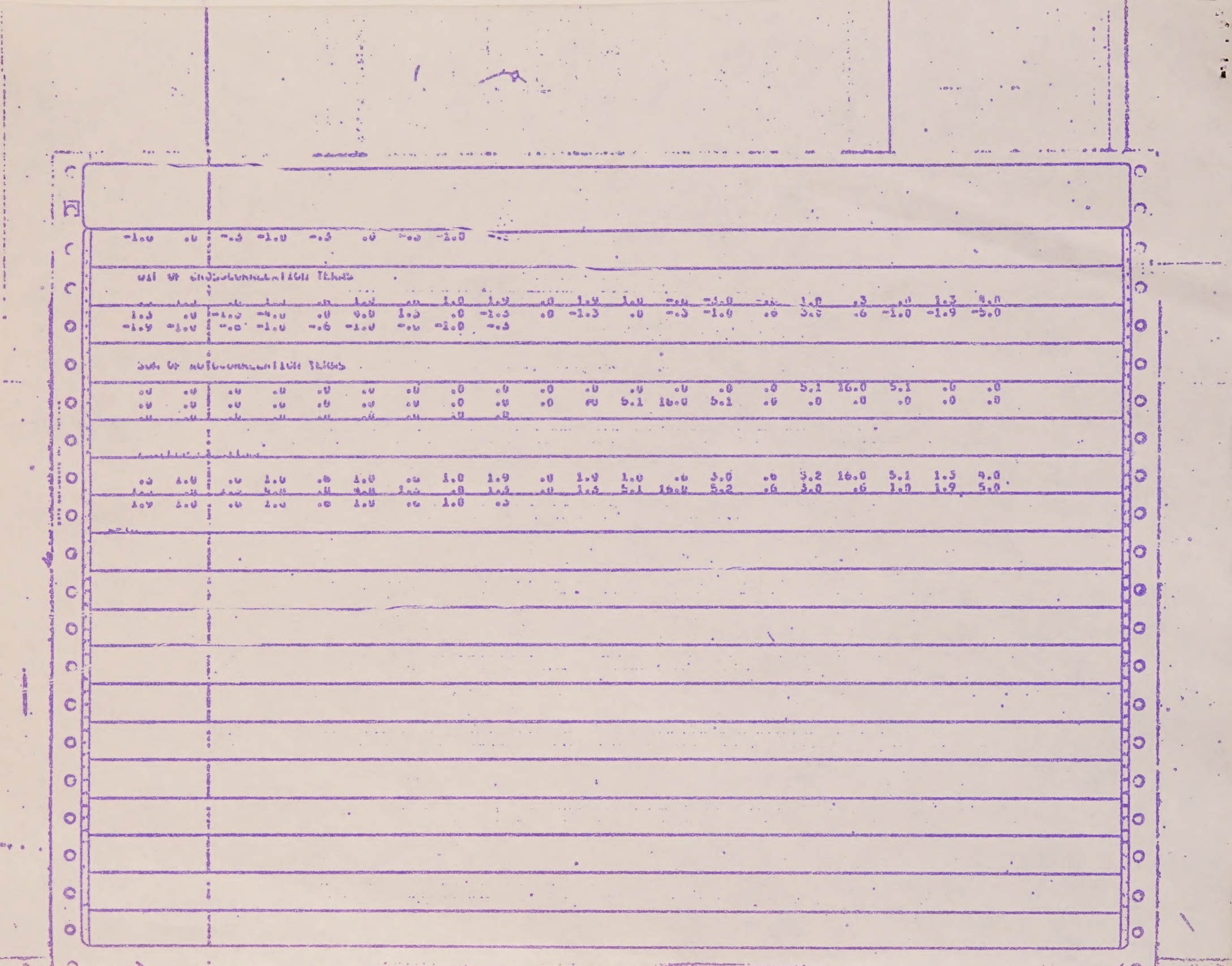
```
      DIMENSION CX(1000),CY(300),CXBAR(  0),CYBAR(300),XXX(1000),XXY(1000
     1),YYY(1000) YYX(1000),AAA(1000),  B(1000),BBB(1000),BBA(1000),
     2)FCRS(100),SUMAUT(1000),ENV(10  )
      COMMON NP1,(200),NP(500),NR(900)
    5 READ(5,10,END=2000) N,(CX(I),I=1  )
      READ(5,10) N,(CY(I),I=1,N)
   10 FORMAT(I;;15F5.0)
      WRITE(7,20)
   20 FORMAT(1H1,'   CODE X',/)
      WRITE(7,25) (CX(I),I=1,N)
   25 FORMAT(20F6.0)
      WRITE(*,30)
   30 FORMAT(1X,/)
      WRITE(7,35)
   35 FORMAT(1X,'   CODE Y',/)
      WRITE(7,40) (CY(I),I=1,N)
```

```
 40 FORMAT(20F6.0)
    WRITE(7,30)
    DO 50 I=1,N
THE TEST FOR EQUALITY BETWEEN NON-INTEGERS MAY NOT BE MEANINGFUL.
    IF(CX(I).EQ.0.) GO TO 45
    CABAR(I)=0.
    GO TO 50
 45 CABAR(I)=1.
 50 CONTINUE
    DO 60 I=1,N
THE TEST FOR EQUALITY BETWEEN NON-INTEGERS MAY NOT BE MEANINGFUL.
    IF(CY(I).EQ.0.) GO TO 55
    CYBAR(I)=0.
    GO TO 60
 55 CYBAR(I)=1.
 60 CONTINUE
    NP=N
    DO 65 I=1,N
    CP(I)=CX(I)
    CP(I+N)=CX(I)
 65 C(I)=CX(I)
    WRITE(7,70)
 70 FORMAT(1X,'   XX/X CORRELATION',/
    CALL CORE
    M=3*N-1
    DO 75 I=1,M
 75 XXX(I)=RR(I)
    DO 80 I=1,N
 80 C(I)=CY(I)
    WRITE(7,85)
 85 FORMAT(1X,'   XX/Y CORRELATION',/
    CALL COREL
    DO 90 I=1,M
 90 XXY(I)=RR(I)
    DO 95 I=1,N
    CP(I)=CY(I)
 95 CP(I+N)=CY(I)
    WRITE(7,100)
100 FORMAT(1X,'   YY/Y CORRELATION',/
    CALL COREL
    DO 105 I=1,M
105 YYY(I)=RR(I)
    DO 110 I=1,N
110 C(I)=CX(I)
    WRITE(7,115)
115 FORMAT(1X,'   YY/X CORRELATION',/
    CALL COREL
    DO 120 I=1,M
120 YYX(I)=RR(I)
    DO 130 I=1,N
    AAA(2*I)=XXX(I)
    AAA(2*I-1)=XXY(I)
    BBB(2*I)=YYY(I)
    BBB(2*I-1)=YYX(I)
    F=0.3183
    AAA(1)=F*AAA(2)
    AAA(1)=F*AAA(2)
```

```
      BBB(1)=F*BBB(2)
      BBA(1)=F*BBA(2)
      MM=M-1
      DO 140 I=1,MM
      J=2*I
      AAA(J+1)=F*(AAA(J)+AAA(J+2))
      AAB(J+1)=F*(AAB(J)+AAB(J+2))
      BBB(J+1)=F*(BBB(J)+BBB(J+2))
  140 BBA(J+1)=F*(BBA(J)+BBA(J+2))
      MMM=2*M
      DO 150 I=1,MMM
      XXX(I+1)=AAA(I)
      XXY(I)=AAB(I)
      YYY(I+1)=BBB(I)
  150 YYX(I+2)=BBA(I)
      XXX(1)=0.
      YYY(1)=0.
      YYX(1)=0.
      YYX(2)=0.
      XXY(MMM+1)=F*XXY(MMM)
      XXY(MMM+2)=0.
      XXX(MMM+2)=F*XXX(MMM+1)
      YYY(MMM+2)=F*YYY(MMM+1)
      YYX(MMM+3)=F*YYX(MMM+2)
      XXX(MMM+3)=0.
      YYY(MMM+3)=0.
      XXY(MMM+3)=0.
      K=MMM+3
      WRITE(7,200)(XXX(I),I=1,K)
      WRITE(7,210)
      WRITE(7,200)(XXY(I),I=1,K)
      WRITE(7,210)
      WRITE(7,200)(YYY(I),I=1,K)
      WRITE(7,210)
      WRITE(7,200)(YYX(I),I=1,K)
      WRITE(7,210)
  200 FORMAT(20F6.1)
  210 FORMAT(1X,/)
      DO 215 I=1,K
      DIFCRS(I)=YYX(I)-XXY(I)
  215 SUMAUT(I)=XXX(I)+YYY(I)
      WRITE(7,220)
  220 FORMAT(1X,'   DIF OF CROSSCORREL  ION TERMS',/)
      WRITE(7,200)(DIFCRS(I),I=1,K)
      WRITE(7,210)
      WRITE(7,230)
  230 FORMAT(1X,'   SUM OF AUTOCORRELA  ON TERMS',/)
      WRITE(7,200)(SUMAUT(I),I=1,K)
      WRITE(7,210)
      DO 300 I=1,K
  300 ENV(I)=SQRT(DIFCRS(I)*DIFCRS(I)+  MAUT(I)*SUMAUT(I))
      WRITE(7,250)
  250 FORMAT(1X,'  ENVELOP MAGNITUDE'  )
      WRITE(7,200)(ENV(I),I=1,K)
      GO TO 5
 2000 CONTINUE
      END
```

EXAMPLE 1 (MAXIMAL-LENGTH)





EXAMPLE 2 (GOLAY'S COMPLEMENTARY)